ML-Based MMSE Separation and Estimation MATLAB Package

This MATLAB Package is provided as supplemental material to the paper:

Amir Weiss and Arie Yeredor, "A Maximum Likelihood-Based Minimum Mean Square Error Separation and Estimation of Stationary Gaussian Sources from Noisy Mixtures", submitted to *IEEE Trans. on Signal Processing*, August 2018.

Notations

We consider the model

$$\boldsymbol{X} = \boldsymbol{A}\boldsymbol{S} + \boldsymbol{V},$$

Where:

- $\boldsymbol{S} \in \mathbb{R}^{M\times T}$ is the matrix of M unobservable "sources";
- $\boldsymbol{A} \in \mathbb{R}^{L\times M}$ is the unknown (deterministic) "mixing" matrix;
- $\boldsymbol{V} \in \mathbb{R}^{L\times T}$ is an additive noise matrix; and
- $\boldsymbol{X} \in \mathbb{R}^{L\times T}$ is the matrix of L observable noisy "mixtures".

and T is the number of measurements. In this model, we further assume the following:

- The sources (i.e., the M rows of $\boldsymbol{S}$) are mutually statistically independent, stationary Gaussian processes with known and distinct (positive-definite) temporal covariance matrices, where $\boldsymbol{C}_s^{(m)}$ is the temporal covariance matrix of $\boldsymbol{s}_m$, the m-th source (i.e., the m-th row of $\boldsymbol{S}$).
- The noise processes (i.e., the L rows of $\boldsymbol{V}$) are mutually statistically independent, temporally-white Gaussian processes with unknown (deterministic) variances, denoted collectively in a vector $\boldsymbol{\lambda} \triangleq \begin{bmatrix} \sigma_{v_1}^2 & \cdots & \sigma_{v_L}^2 \end{bmatrix}$, where $\sigma_{v_\ell}^2 \in \mathbb{R}^+$ is the variance of the ℓ-th row of $\boldsymbol{V}$.
- The mixing matrix $\boldsymbol{A}$ is full rank (and $L \geq M$).

Content

This package contains two files (not including this instruction file):

1. `Compute_sources_ML_MMSE_estimate.m`
2. `Script_ML_MMSE.m`

Description

1. `Compute_sources_ML_MMSE_estimate.m` – This function computes the ML-based MMSE estimate of the sources, and is implemented *exactly* according to the pseudo-code which appears as "Algorithm 1" in the paper. Accordingly, the input is simply the mixtures matrix $\boldsymbol{X}$ and the M temporal covariance matrices of the sources $\left\{\boldsymbol{C}_s^{(m)}\right\}_{m=1}^{M}$. We note that while the input for the mixtures is an L x T (real-valued) matrix, since the sources are assumed to be stationary, each matrix $\boldsymbol{C}_s^{(m)}$ may be represented (compactly) by the autocorrelation sequence of the m-th sources. Therefore, this input is given in the form of an M x T matrix, denoted by `C_s(:,:)`, such that the m-th row is the centralized autocorrelation sequence of the m-th source. In particular, if we denote by `C_s_m(:,:)` the T x T temporal covariance matrix of the m-th source, then, with our notations,

   ```
   C_s(m,:) = C_s_m(T/2+1,:),
   ```

   where we assume T is even, for simplicity.

   The output of this function is the ML-based MMSE estimate of the sources $\boldsymbol{S}$, as well as the ML estimates of $\boldsymbol{A}$ and $\boldsymbol{\lambda}$.[1]

   Note: The file contains additional (necessary) local functions, so it is self-contained. For more details see the in-code documentation in the file.

[1] Strictly speaking, the output is only a *candidate* of the ML estimates of the quantities above, since it is computed numerically via the (iterative) Fisher scoring algorithm, which converges, in general, to a stationary point of the likelihood, not necessarily being the global maximizer. However, from our experience, with the proposed initial estimates, it converges to the ML estimates w.h.p.

2. `Script_ML_MMSE.m` – this script demonstrates the operation of the function `Compute_sources_ML_MMSE_estimate` for first-order autoregressive Gaussian sources. It also measures the overall running time of the function (stored in the variable `running_time`).

   For more details see the in-code documentation in the file.