

The Extended “Sequentially Drilled” Joint Congruence Transformation and its Application in Gaussian Independent Vector Analysis

MATLAB Package

Notations

We consider the model

$$\mathbf{X}^{(m)} = \mathbf{A}^{(m)} \mathbf{S}^{(m)}, \quad \forall m \in \{1, \dots, M\},$$

where:

- $\mathbf{S}^{(m)} \in \mathbb{R}^{K \times T}$ is the matrix of K unobservable “sources”;
- $\mathbf{A}^{(m)} \in \mathbb{R}^{K \times K}$ is the unknown (deterministic) “mixing” matrix; and
- $\mathbf{X}^{(m)} \in \mathbb{R}^{K \times T}$ is the matrix of K observable “mixtures”,

with T as the number of measurements. Denote $\mathbf{S}^{(m)} \triangleq [\mathbf{s}_1^{(m)} \dots \mathbf{s}_K^{(m)}]^\top \in \mathbb{R}^{K \times T}$.

In this model, we assume the following:

- The sources *within each dataset* are mutually statistically *independent*:

$$\mathbf{s}_{k_1}^{(m_1)} \perp \mathbf{s}_{k_2}^{(m_2)}, \quad \forall k_1 \neq k_2 \in \{1, \dots, K\}, \forall m_1 \neq m_2 \in \{1, \dots, M\},$$

where \perp denotes statistical independence.

- Statistical dependence between respective sources from different datasets is considered, namely the vector $\mathbf{s}_k^{(m_1)}$ may depend on the vector $\mathbf{s}_k^{(m_2)}$.
- The sources are zero-mean Gaussian processes with known and distinct (positive-definite) temporal covariance matrices $\mathbf{C}_k^{(m_1, m_2)} \triangleq E[\mathbf{s}_k^{(m_1)} \mathbf{s}_k^{(m_2)\top}] \in \mathbb{R}^{T \times T}$, i.e., $\mathbf{C}_k^{(m_1, m_2)}$ is the temporal covariance matrix between the k -th source at the m_1 -th dataset and the k -th source at the m_2 -th dataset.
- The mixing matrices $\{\mathbf{A}^{(m)}\}_{m=1}^M$ are all invertible.

Content

This package contains four files (not including this instruction file):

1. `Solve_Extended_SeDJoCo_Newton.m`
2. `Solve_Extended_SeDJoCo_IR.m`
3. `Script_Extended_SeDJoCo.m`
4. `Script_separate_stationary_sources.m`

Description

1. `Solve_Extended_SeDJoCo_Netwon.m` — This function computes the solution of the extended SeDJoCo problem (Subsection II-A, Problem P1, eq. (23) in [1]) by Newton's method (Subsection IV-B in [1]).

The input is a set of KM^2 target matrices $\{Q_k^{(m_1, m_2)} \in \mathbb{R}^{K \times K}\}$ target-matrices, as defined in eq. (16) in [1]. The output is a set of M matrices $\{B^{(m)} \in \mathbb{R}^{K \times K}\}$, solving the corresponding extended SeDJoCo problem. In the context of our IVA problem, these are the ML estimates of the separating matrices $\left\{ \left(A^{(m)} \right)^{-1} \in \mathbb{R}^{K \times K} \right\}$.

Note: The file contains additional (necessary) local functions, so it is self-contained. For more details see the in-code documentation in the file.

2. `Solve_Extended_SeDJoCo_IR.m` — This function computes the solution of the extended SeDJoCo problem (Subsection II-A, Problem P1, eq. (23) in [1]) by IR (Subsection IV-A in [1]).

The input is a set of KM^2 target matrices $\{Q_k^{(m_1, m_2)} \in \mathbb{R}^{K \times K}\}$ target-matrices, as defined in eq. (16) in [1]. The output is a set of M matrices $\{B^{(m)} \in \mathbb{R}^{K \times K}\}$, which are the ML estimates of the separating matrices $\left\{ \left(A^{(m)} \right)^{-1} \in \mathbb{R}^{K \times K} \right\}$.

Note: The file contains an additional (necessary) local function, so it is self-contained. For more details see the in-code documentation in the file.

3. `Script_Extended_SeDJoCo.m` — this script demonstrates the operation of `Solve_Extended_SeDJoCo_IR` and `Solve_Extended_SeDJoCo_Newton` for a generic extended SeDJoCo problem. The target-matrices are generated according to the description in Subsection V-A in [1]. For more details see the in-code documentation in the file.
4. `Script_separate_stationary_sources.m` — this script demonstrates the separation of stationary sources within the framework of IVA, according to the model above, by solving the likelihood equations, namely the associated extended SeDJoCo problem. Here, the target-matrices are computed using the mixtures and the temporal auto- and cross-covariance matrices (in the frequency domain). At the end of the run, the estimated global “demixing-mixing” matrices are displayed.

References:

- [1] Weiss, A., Yeredor, A., Cheema, S. A. and Haardt, M., “The Extended “Sequentially Drilled” Joint Congruence Transformation and its Application in Gaussian Independent Vector Analysis”, *IEEE Transactions on Signal Processing*, vol. 65, no. 23, pp. 1-13, Dec. 2017.