Estimation, Filtering and Decoding via Deep Learning "Cheat Sheet" | Amir Weiss, Alejandro Lancho and Gary Lee

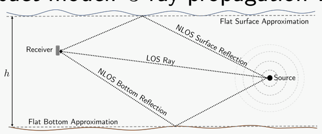
Optimal model-based solution and analysis → Extraction of key domain knowledge → Architecture design for a given task

Phase 1: Simplified Model-Based Approach

1. **Devise "compact" model amenable for analysis**
2. **Solve optimally for a chosen optimality criterion**
3. **Analyze the solution and derive performance bounds**

Example 1: Underwater Acoustic Localization

1. Devise compact model: 3-ray propagation with AWGN [1]



2. Solve optimally: Least-squares estimator [1]

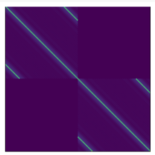
$$\widehat{\mathbf{p}} = \underset{\boldsymbol{p}\in\mathbb{R}^{3\times 1}}{\arg\max}\ \lambda_{\max}\left(\underbrace{\mathbf{Q}(\boldsymbol{p},\mathbf{x}_1,\ldots,\mathbf{x}_L)}_{\text{Position- and data-dependent matrix}}\right)$$

3. Analyze performance: Cramér-Rao lower bound

Example 2: Interference Rejection in RF Digital Communication Systems

1. Devise compact model: Gaussian cyclostationary [2]

Temporal covariance of an OFDM signal



2. Solve optimally: MMSE and MAP-QLMMSE [3]

$$\widehat{\mathbf{s}}_{\text{MMSE}} = \sum_{m_s=1}^{K_s}\sum_{m_b=1}^{K_b}\Pr\left(\mathrm{k}_s = m_s, \mathrm{k}_b = m_b|\mathbf{y}\right)\widehat{\mathbf{s}}_{\text{LMMSE}}(m_s,m_b)$$

$$\widehat{\mathbf{s}}_{\text{MAP-QLMMSE}} \triangleq \widehat{\mathbf{s}}_{\text{LMMSE}}\left(\widehat{\mathrm{k}}_b^{\text{MAP}}\right)$$

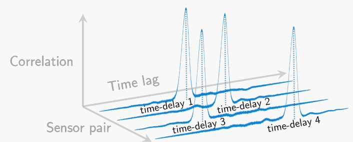
3. Performance bound: MMSE

Phase 2: Key Domain Knowledge

1. **Identify key signal statistics (e.g., sufficient statistic)**
2. **Pinpoint weaknesses of optimal model-based solution**
3. **Select learning type based on measure of "goodness"**

Example 1: Underwater Acoustic Localization

1. Sufficient statistics: empirical auto- cross-**correlations** [4]

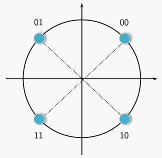


2. Model-based solution weaknesses: **High computational complexity** (eigenvalue problem for each point on the grid, nonlinear (and nonconvex) optimization), **sensitive to model mismatch**, **impractical for general bathymetry**
3. Learning type: Measure of goodness → MSE ⟹ Regression

Example 2: Interference Rejection in RF Digital Communication Systems

1. Key signal statistics: cyclostationarity → cyclic periods

OFDM → FFT size [5]
and
underlying **discrete "nature"** (alphabet)



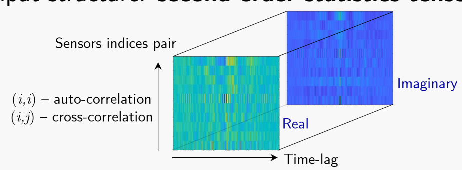
2. Model-based solution weaknesses: **High computational complexity** (inversion of large matrices, brute-force search over all possible sequences), **Does not easily generalize when channel impairments are considered**
3. Learning type: Measure of goodness → MSE / BER ⟹ Regression / Classification

Phase 3: Architecture Design

1. **Determine input structure informed by key statistics**
2. **Choose/Develop NN main structure / building block**
3. **Set key hyperparameters and training procedure**
4. **Select loss functions that are matched to estimands**

Example 1: Underwater Acoustic Localization

1. Input structure: **second-order statistics tensor** [4]



2. Main building block: Conv2D → joint filtering of all correlation functions simultaneously
3. Key hyperparameters: **kernel size** of first layer → **maximum estimated time-delay** between different sensors
Training procedure: (i) per coordinate → (ii) 3D position
4. Loss functions: 3D position → squared-error in spherical coordinates (**gradients** of different coordinates **numerically dependent**, **periodic in azimuth/inclination** angles)

Example 2: Interference Rejection in RF Digital Communication Systems

1. Input structure: separated real and imaginary parts of the input mixture (widely linear estimation)
2. Chosen structure: UNet [3, 5]—down- and up-sampling operations (filtering at different temporal resolutions)
3. Key hyperparameters: **kernel size** of first layer → **capture nonlocal correlations** (e.g., cyclic prefix of OFDM)
4. Loss functions: source **separation** → **squared-error**
signal-of-interest **demodulation** → **cross-entropy**

References

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[2] G. C. Lee, A. Weiss, A. Lancho, J. Tang, Y. Bu, Y. Polyanskiy, and G. W. Wornell, “Exploiting temporal structures of cyclostationary signals for data-driven single-channel source separation,” in *Proc. IEEE 32nd Int. Workshop Mach. Learn. Signal Process. (MLSP)*, 2022, pp. 1–6.

[3] A. Lancho, A. Weiss, G. C. Lee, J. Tang, Y. Bu, Y. Polyanskiy, and G. W. Wornell, ,” in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM)*.

[4] A. Weiss, T. Arikan, and G. W. Wornell, “Direct localization in underwater acoustics via convolutional neural networks: A data-driven approach,” in *Proc. IEEE 32nd Int. Workshop Mach. Learn. Signal Process. (MLSP)*, 2022, pp. 1–6.

[5] G. C. Lee, A. Weiss, A. Lancho, Y. Polyanskiy, and G. W. Wornell, ,” in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*.

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